

M3PM16/M4PM16 PROBLEMS 5. 12.2.2015

Q1. By considering the series expansion of $-\log(1-x)$, or otherwise, show that $\prod(1-1/p)$ diverges.

Q2. Use the divergence of $\prod(1-1/p)$ to show (by considering the number $N(x, r)$ of $n \leq x$ not divisible by any of the first r primes p_k , or otherwise) that

$$\pi(x) = o(x).$$

(This bound is much weaker than PNT $\pi(x) \sim li(x) \sim x/\log x$, but is useful and non-trivial.)

Q3. Show that if $c := a * b$ and b are multiplicative, then a is multiplicative.

Q4. Show that

$$d * \mu = \delta,$$

and give the corresponding identity for Dirichlet series.

Q5. Show that the divisor function d has sum-function

$$D(x) := \sum_{n \leq x} d(n) = x \log x + O(x).$$

(This question is for completeness only: we got a better result by DHI in II.9 L14.)

NHB