m3pm16prob8(14).tex

M3PM16/M4PM16 PROBLEMS 8. 12.3.2015

Q1 *Sum-function of ϕ.* Show that

$$\sum_{n \le x} \phi(n) = \frac{3}{\pi^2} x^2 + O(x \log x).$$

Q2 *$Q(x)$ and PNT.*
   Given PNT in the form

$$M(x) := \sum_{n \le x} \mu(n) = o(x),$$

show that we can improve the error term $O(.)$ in Problems 7 Q4(iii) to $o(.)$:

$$Q(x) = \frac{6}{\pi^2} x + o(\sqrt{x}).$$

(Use Problems 7 Q3 and Dirichlet's hyperbola identity with $a = 1$, $b = \nu$, letting first $x \to \infty$ and then $y \to \infty$.)
Note. One can prove more, from PNT with remainder (Ch. IV): $o(\sqrt{x})$ can be improved to $O(x \exp\{-\frac{1}{2} c \sqrt{\log x}\})$, where $O(x \exp\{-c\sqrt{\log x}\})$ is the remainder term in PNT (MV, 6.2.1 Ex. 19(d) p.186).

NHB

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