m3pm16prob9.tex

M3PM16/M4PM16 PROBLEMS 9. 19.3.2015

Q1: ξ and ζ.

Show that:

(i) For large $|s|$,

$$|\zeta(s)| = O(|s|).$$

(ii) ξ is an entire function of order 1.

(iii) For some constants A, B,

$$\xi(s) = e^{A+Bs} \prod_{\rho}(1 - s/\rho)e^{s/\rho},$$

where $\rho = \beta + i\gamma$ runs over the zeros of the zeta function ζ.

Q2 *Partial fraction expansions.*

In Q1(iii), obtain the partial fraction expansions

$$\frac{\xi'(s)}{\xi(s)} = B + \sum_{\rho}\left(\frac{1}{s - \rho} + \frac{1}{\rho}\right),$$

$$-\frac{\zeta'(s)}{\zeta(s)} = -B + \frac{1}{s - 1} - \frac{1}{2}\log\pi + \frac{1}{2}\frac{\Gamma'(\frac{1}{2}s + 1)}{\Gamma(\frac{1}{2}s + 1)} - \sum_{\rho}\left(\frac{1}{s - \rho} + \frac{1}{\rho}\right).$$

NHB