m3a22prob6.tex

PROBLEMS 6. 20.11.2015

Q1. *The doubling strategy.*

Analyze the 'doubling strategy' ("the original martingale"): when betting on tossing a fair coin, respond to losing by doubling the stakes.
(i) Show that an eventual win is certain.
(ii) Show that this certain gain is 1.
(iii) Show that the mean time to this win is 1.
(iv) Explain why this is an impossible (or at least suicidal) strategy to follow in practice.
Note. 1. It ought not to be possible to 'make something out of nothing' in this way – and it isn't.
2. There are those who respond to confrontation by 'upping the ante' in a similar way. You might care to ponder the implications of this as a life strategy.

Q2. *Simple random walk (SRW).*

We bet on independent tosses of a fair coin. Our strategy is to bet until we are first ahead, and then quit. Let T be the time we quit (a stopping time). Show that:
(i) T has PGF

$$E[s^T] = \frac{1 - \sqrt{1 - s^2}}{s}$$

(hint: look at what happens if we play until we are 2 ahead for the first time);
(i) $T < \infty$ a.s.: we are certain of achieving a net gain of $+1$ eventually;
(ii) $ET = +\infty$: the mean waiting time till this happens is infinite;
(iii) this strategy too is impossible in practice.

Recommended Reading: Grimmett & Stirzaker [GS], §5.2 (Bingham & Kiesel Ex. 3.4).

NHB