M3A22/M4A22/M5A22 EXAMINATION 2015-16

Q5 is the Mastery Question, for M4A22/M5A22 candidates only.

Q1. Comment briefly on:
(a) limited liability;
(b) moral hazard;
(c) liquidity;
(d) size of traders.

Q2. In a (Cox-Ross-Rubinstein) binomial-tree model with discount rate $1+\rho$ per period, 'up' and 'down' factors $1+u$, $1+d$ and 'up' and 'down' probabilities q, $1-q$, find the condition for q to be the risk-neutral probability.

Describe how to price an American put with strike K in an N-period binomial-tree model.

What is the connection here with the Snell envelope?

Q3. (a) Give the stochastic differential equation for $S = (S_t)$ geometric Brownian motion $GBM(\mu, \sigma)$ with parameters μ and σ. State its solution, without proof.
(b) State the risk-neutral valuation formula (in continuous time), applied to a European call option with stock price S_t at time $t \in [0, T]$, strike price K, riskless interest rate r, volatility σ and expiry T.
(c) Hence or otherwise derive the Black-Scholes formula for the price of the call at time $t = 0$:

$$c_0 = S_0 \Phi(d_+) - K e^{-rT} \Phi(d_-), \quad d_\pm := [\log(S_0/K) + (r \pm \frac{1}{2}\sigma^2)T]/\sigma\sqrt{T}. \quad (BS)$$

Q4. (a) Formulate the problem of real options as an optimal-stopping problem.
(b) Show that we may restrict to the case $0 < \mu < r$, where μ is the mean return on the investment and r is the riskless interest rate.
(c) Obtain the fundamental quadratic equation, with roots $p_2 < 0$, $1 < p_1$.
(d) Show that one should not invest the necessary capital I unless the initial value is at least qI, where $q := p_1/(p_1 - 1) > 1$.
(e) Why do arbitrage arguments play no role here?

Q5. (i) Define the Sharpe ratio.
(ii) Describe briefly, without proof, how to derive the Black-Scholes formula

in continuous time from Girsanov's theorem.
(iii) Obtain a hedging strategy for the options under the Black-Scholes model in continuous time. Comment on the practical value of this result.

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