m3f33prob1

M3F22 PROBLEMS 1. 13.10.2017

Q1 *(Random sums; Wald's identity).*

If the X_n are independent and identically distributed (iid), non-negative integer-valued with PGF $P(s)$, and N is non-negative integer-valued with PGF $Q(s)$ and independent of the X_n, show that the random sum

$$Y := X_1 + \cdots + X_N$$

has PGF

$$R(s) = Q(P(s)).$$

Deduce (a form of Wald's identity) that

$$E[Y] = E[X_1 + \cdots + X_N] = E[X_1].E[N].$$

(Context: Insurance, Ch. VIII; Xs the claim sizes, N the number of claims.)

Q2 (*Compound Poisson distribution*). If N is Poisson $P(\lambda)$ and the X_n are iid (not necessarily integer-valued) with CF $\phi(t)$, mean μ and variance σ^2, show that Y has CF

$$\psi(u) = \exp\{-\lambda(1 - \phi(u))\},$$

mean $\lambda\mu$ and variance $\lambda E[X^2] = \lambda(\mu^2 + \sigma^2)$.

Q3 (*Compound Poisson process*).

Show that if claims X_n arrive in a Poisson process of rate λ, and have CF, mean and variance as above, the claim total at time t has CF, mean and variance

$$\psi(u) = \exp\{-\lambda t(1 - \phi(u))\}, \qquad \lambda t\mu, \qquad \lambda t E[X^2] = \lambda t(\mu^2 + \sigma^2).$$

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