m3f33prob3

M3F22 PROBLEMS 3. 27.10.2017

Q1. (i) Show that the volume of a sphere of radius r is $V = 4\pi r^3/3$.
(ii) Show that the surface area of a sphere of radius r is $S = 4\pi r^2$.
(iii) Derive each from the other.

Q2. Show that the volume of the ellipsoid

$$x^2/a^2 + y^2/b^2 + z^2/c^2 = 1$$

is $V = 4\pi abc/3$.

Q3. (i) Show that the volume of a tetrahedron of base area A and height h is $V = Ah/3$.
(ii) Show that this holds also for 'generalised tetrahedrons', obtained by taking any plane shape with area A and boundary curve C, and joining the points of C to some vertex V a height h above C.

Q4. (i) Show that the volume of revolution obtained by rotating a curve $y = f(x)$ about the x-axis between a and b is $V = \pi \int_a^b f(x)^2 dx$.
(ii) Hence re-derive the volume of a sphere.

Q5 (*Generalised Pythagoras theorem: Bouligand*). A right-angled triangle has sides 1 (the hypotenuse), 2 and 3. A semicircle (or any other plane shape with a flat base) of area A_1 is drawn with base side 1; similar copies of this are drawn with bases sides 2 and 3, with areas A_2, A_3. Show that

$$A_1 = A_2 + A_3.$$

Deduce Pythagoras' theorem on taking these shapes to be squares.

NHB