

MPC2: ASSESSED COURSEWORK 2011

Wed 30 November; deadline Mon 12 December

Q1. For the non-homogeneous equation

$$y'' - 2y'/x + 2y/x^2 = xe^x \quad (x > 0), \quad (N)$$

(i) show that $y_1 = x$ and $y_2 = x^2$ are solutions of the corresponding homogeneous equation (H); [4,4]

(ii) find the general solution to (N). [17]

Q2. (i) Show that $\cos n\theta$ is a polynomial of degree n in $\cos \theta$, $\cos n\theta = T_n(\cos \theta)$ say. [10]

(ii) Find T_7 . [15]

Q3. Show that (working in spherical polar coordinates)

(i) $f(r + ct)/r$ is a solution of the wave equation

$$\Delta u = u_{tt}/c^2, \quad [15]$$

(ii) $g(r - ct)/r$ is also a solution for $t < r/c$. [5]

(iii) Interpret these results. [5]

Q4. For the matrix

$$A = \begin{pmatrix} 3 & 2 & 2 \\ 1 & 2 & 2 \\ -1 & -1 & 0 \end{pmatrix},$$

find the eigenvalues [4,4,4]. For each eigenvalue,

(i) state its multiplicity [3];

(ii) give as many linearly independent eigenvectors as possible, and state the dimensionality of the eigenspace [10] .

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