

PROBLEMS 1. 12.10.2011

Q1. For $x > 0$,

(i) Find two independent solutions of the homogeneous DE

$$x^2 y'' - 2y = 0 \quad (H)$$

of the form $y = x^\lambda$.

(ii) Hence find the general solution of the non-homogeneous DE

$$x^2 y'' - 2y = 2x^5. \quad (N)$$

(iii) Check your working: verify that your answer does satisfy (N).

Q2. Show that the general solution of

$$y'' + \lambda^2 y = f(x)$$

with ICs $y(0) = y_0$, $y'(0) = y_1$ is

$$y(x) = \frac{1}{\lambda} \int_0^x f(t) \sin \lambda(x-t) dt + y_0 \cos \lambda x + \frac{y_1}{\lambda} \sin \lambda x.$$

(Hint: Use the method of lectures.)

Q3. Find the general solution of

$$y'' - 5y' + 6y = 2e^x.$$

(Hint: Use trial solutions $e^{\lambda x}$.)

NHB