mpc2prob2.tex

PROBLEMS 2. 19.10.2011

Q1. Check that

$$\begin{aligned} G(x,y) &= \frac{\sin\ ky\ \sin\ k(\ell-x)}{k\sin\ k\ell} \qquad (0\le y\le x) \\ &= \frac{\sin\ kx\ \sin\ k(\ell-y)}{k\sin\ k\ell} \qquad (x\le y\le \ell) \end{aligned}$$

is a Green function for

$$u''+k^2u=-f(x) \qquad (0\le x\le \ell), \qquad u(0)=u(\ell)=0$$

– i.e., it has the properties of Lectures.

Q2. Prove from first principles that $\cos(\pi/3)=1/2$.

Q3. Express the following quotients in standard form (i.e., rationalize the denominators):

$$\frac{3+i}{2+i}; \qquad \frac{4+3i}{3+4i}.$$

Q4. Solve the simultaneous equations

$$\begin{aligned} u+v &= 10, \\ uv &= 40, \\ u,v & \text{ a conjugate complex pair.} \end{aligned}$$

Q5. Solve the cubic equation

$$x^3=9x+28.$$

Hint: (i) Find by inspection a root of $x^3-9x-28$.
(ii) Hence factor this cubic into a linear factor times a quadratic factor, and
(iii) Solve this quadratic.

NHB