

PROBLEMS 3. 26.10.2011

Q1. Express each of the following in polar form:

- (i) i ;
- (ii) $1 - i$;
- (iii) $\sqrt{3} - i$;
- (iv) $(1 - i)/(\sqrt{3} - i)$.

Q2. Show that

- (i) $\sin(A + B) + \sin(A - B) = 2 \sin A \cos B$;
- (ii) $\sin(A + B) - \sin(A - B) = 2 \cos A \sin B$;
- (iii) $\cos(A + B) + \cos(A - B) = 2 \cos A \cos B$;
- (iv) $\cos(A + B) - \cos(A - B) = 2 \sin A \sin B$.

Q3. Let v, w be the solutions to the wave equation

$$y_{xx} = c^{-2}y_{tt} \quad (WE)$$

with BCs $y(0, t) = 0, y(\ell, t) = 0$ and ICs

- (i) $v(x, 0) = f(x), v_t(x, 0) = 0$ ($0 \leq x \leq \ell$),
- (ii) $w(x, 0) = 0, w_t(x, 0) = g(x)$ ($0 \leq x \leq \ell$)

respectively. Let u be the solution satisfying the same BCs but with ICs

$$u(x, 0) = f(x), \quad u_t(x, 0) = g(x).$$

Show that

$$u = v + w.$$

Q4. Show that the solution to the wave equation (WE) above satisfying the ICs

$$u(x, 0) = h(x), \quad u_t(x, 0) = 0$$

is

$$y = \frac{1}{2}[h(x + ct) + h(x - ct)].$$

NHB