mpc2prob4.tex

PROBLEMS 4. 2.11.2011

Q1. Show that the solution to the wave equation $y_{xx} = c^{-2} y_{tt}$ with ICs

$$y(x,0) = 0, \qquad y_t(x,0) = h(x)$$

is

$$y(x,t) = \frac{1}{2c} \int_{x-ct}^{x+ct} h(u) du.$$

Q2. Hence or otherwise solve the wave equation with ICs

$$y(x,0) = h_1(x), \qquad y_t(x,0) = h_2(x).$$

Q3. Show that

$$u = u(x,t) = \frac{1}{2\sqrt{\pi k t}} \exp\{-(x-s)^2/(4kt)\}$$

satisfies the heat equation $u_{xx} = u_t/k$.

Q4. Deduce that

$$u = \frac{1}{2\sqrt{\pi k t}} \int_{-\infty}^{\infty} f(s) \exp\{-(x-s)^2/(4kt)\} ds$$

also satisfies the heat equation.

Note. 1. We shall see later that u in Q4 also satisfies the IC

$$u(x,0) := \lim_{t \downarrow 0} u(x,t) = f(x).$$

2. u in Q3 is called the *fundamental solution* of the heat equation at the point $x = s$.

NHB