

PROBLEMS 7. 23.11.2011

Q1. Show that

$$A = \begin{pmatrix} 3 & 2 & 4 \\ 2 & 0 & 2 \\ 4 & 2 & 3 \end{pmatrix}$$

has

(i) a simple eigenvalue 8 with eigenvector

$$\begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix},$$

(ii) a double eigenvalue -1 with eigenvectors

$$\begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix}, \quad \begin{pmatrix} 0 \\ -2 \\ 1 \end{pmatrix}.$$

Q2. Let P be the matrix of eigenvectors and D the diagonal matrix of eigenvalues here:

$$P := \begin{pmatrix} 2 & 1 & 0 \\ 1 & -2 & -2 \\ 2 & 0 & 1 \end{pmatrix}, \quad D := \begin{pmatrix} 8 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}.$$

Find P^{-1} , and check that

$$P^{-1}AP = D.$$

NHB