mpc2soln1.tex

SOLUTIONS 2. 24.10.2011

Q1. (i) G is continuous;
(ii) Using prime for $\partial/\partial x$ (differentiation w.r.t. the first argument),

$$\begin{aligned} G'(x,y) &= -\frac{k\sin\ ky\ \cos k(\ell-x)}{k\ \sin\ k\ell} \qquad (0 \le y < x), \\ &= \frac{k\cos\ kx\ \sin k(\ell-y)}{k\ \sin\ k\ell} \qquad (x < y \le \ell). \end{aligned}$$

Let $x \downarrow y$:

$$G'(y+,y) = \frac{\sin\ ky\ \cos k(\ell-y)}{\sin\ k\ell}.$$

Let $x \uparrow y$:

$$G'(y-,y) = \frac{\cos\ ky\ \sin k(\ell-y)}{\sin\ k\ell}.$$

So

$$\begin{aligned} G'(y+,y) - G'(y-,y) &= -\frac{[\sin\ ky\ \cos\ k(\ell-y) + \cos\ ky\ \sin\ k(\ell-y)]}{\sin\ k\ell} \\ &= -\frac{\sin\ k\ell}{\sin\ k\ell} = -1. \end{aligned}$$

(iii)

$$\begin{aligned} G''(x,y) &= -k\frac{\sin\ ky\ \sin\ k(\ell-x)}{\sin\ k\ell} \qquad (0 \le y < x), \\ &= -k\frac{\sin\ kx\ \sin\ k(\ell-y)}{\sin\ k\ell} \qquad (x < y \le \ell) \\ &= -k^2 G(x,y). \end{aligned}$$

Combining, G is the required Green function, as it has the required properties.

Q2. Draw an equilateral triangle ABC with unit sides. Drop the perpendicular from B to AC. This bisects AC, by symmetry, at D say. So ABD is a right-angled triangle with other angles $\pi/3$, $\pi/6$ (or 30-60-90 in degrees). Angle DAB, which is $\pi/3$ by symmetry (the three angles of ABC sum to π)

has cosine $AD/AB = 1/2$, by definition of cosine.

Q3. (i)

$$\frac{3+i}{2+i} = \frac{(3+i)(2-i)}{(2+i)(2-i)} = \frac{(6+1)+i(-3+2)}{4+1} = \frac{7-i}{5}.$$

(ii)

$$\frac{4+3i}{3+4i} = \frac{(4+3i)(3-4i)}{(3+4i)(3-4i)} = \frac{(12+12)+i(9-16)}{9+16} = \frac{24-7i}{25}.$$

Q4. Write

$$z = x + iy = u, \qquad \bar{z} = x - iy = v.$$

Then

$$u + v = 2x = 10, \qquad x = 5,$$

$$uv = z\bar{z} = |z|^2 = x^2 + y^2, \qquad 40 = 25 + y^2, \quad y^2 = 15, \quad y = \sqrt{15}.$$

So $u, v = 5 \pm i\sqrt{15}$.

Q5. (i) After a few trial values, one finds that the given cubic $p(x)$ has $p(4) = 0$. So it has $x - 4$ as a factor.
(ii) Then

$$p(x) := x^3 - 9x + 28 = (x-4)(x^2 + 4x + 7)$$

(the constant term $+7$ is clear on comparing constant terms left and right; that the coefficient a of x in the quadratic is 4 comes from comparing x terms left and right: $7 - 4a = -9$.
(iii) The quadratic has roots $-2 \pm i\sqrt{3}$, so the roots of $p(x)$ are 4, $-2 \pm i\sqrt{3}$.

NHB