mpc2soln3.tex

**SOLUTIONS 3. 31.10.2011**

Q1. Draw an Argand diagram in each case.
(i) $i = e^{i\pi/2}$.
(ii) $1 - i = \sqrt{2}e^{-i\pi/4} = \sqrt{2}e^{7i\pi/4}$.
(iii) $\sqrt{3} - i = 2e^{-i\pi/6}$.
(Recall – see e.g Problems 2 – that $\cos(\pi/3) = \sin(\pi/6) = 1/2$, so $\sin(\pi/3) = \cos(\pi/6) = \sqrt{3}/2$, so $\tan(\pi/6) = 1/\sqrt{3}$.)
(iv) By (ii) and (iii),

$$\frac{(1-i)}{(\sqrt{3}-i)} = \frac{\sqrt{2}e^{-i\pi/4}}{2e^{-i\pi/6}} = \frac{1}{\sqrt{2}}e^{-i\pi/12}.$$

Q2. These follow from the trigonometric addition formulae in lectures, by
(i) adding $\sin(A \pm B)$;
(ii) subtracting $\sin(A \pm B)$;
(iii) adding $\cos(A \pm B)$;
(iv) subtracting $\cos(A \pm B)$.
*Note.* These formulae will be important in Ch. V on Fourier series, where we will use them to derive the orthogonality of sines and cosines, and of distinct sines and distinct cosines.

Q3. If $U := v + w$,
(i) $U$ satisfies the wave equation $(WE)$, by linearity;
(ii) $U$ satisfies the BCs (vanishes for all $t$ at $x = 0, \ell$) as $v$, $w$ do;
(iii) satisfies the ICs, as

$$U(x,0) = v(x,0) + w(x,0) = f(x) + 0 = f(x),$$

$$U_t x,0) = v_t(x,0) + w_t(x,0) = 0 + g(x) = g(x).$$

So $U = v+w$, $u$ satisfy the same PDE, BCs and ICs, so they agree: $u = v+w$.

Q4. As in lectures, $h(x \pm ct)$ satisfy the wave equation. So by linearity,

$$y := \frac{1}{2}[h(x+ct) + h(x-ct)]$$

also satisfies the wave equation. Clearly $y(x,0) = \frac{1}{2}[h(x) + h(x)] = h(x)$. Also

$$y_t = \frac{c}{2}[h'(x+ct) - h'(x-ct)],$$

so $y_t(x,0) = \frac{c}{2}[h'(x) - h'(x)] = 0$. So $y$ satisfies the PDE and ICs, as required.

NHB