mpc2soln5.tex

SOLUTIONS 5. 14.11.2011

Q1. Number the equations (1) – (3). Then [(1)-(2), to eliminate x_1, and (3)]

$$\begin{aligned} 4x_2 - 5x_3 &= 3, \\ -3x_2 + 4x_3 &= -2. \end{aligned}$$

Call these (i) and (ii). Then [3(i) + 4(ii), to eliminate x_2]: $x_3 = 1$.
Back-substitute in (i): $4x_2 = 8$, $x_2 = 2$.
Back-substitute in (1): $x_1 = 9 - 10 - 2 = -3$, giving

$$x_1 = -3, \quad x_2 = 2, \quad x_3 = 1.$$

[Check!]

Q2. $|A| = 1(4+21) - 5(4) + 2(-3) = 25 - 20 - 6 = -1$.

Q3.

$$AB = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I,$$

$$BA = \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & \\ 0 & 1 \end{pmatrix} = I$$

(so $B = A^{-1}$, the inverse of A. We can see this, as $|A| = 1$ and B is the transposed matrix of cofactors of A.)

$$DB = \begin{pmatrix} 4 & 0 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 4 & -4 \\ 2 & -1 \end{pmatrix},$$

$$AC = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 3 \\ 2 & 1 \end{pmatrix} = \begin{pmatrix} 3 & 4 \\ 2 & 1 \end{pmatrix}.$$

Q4.

$$BE = \begin{pmatrix} 1 & 2 & 1 \\ 1 & 3 & 2 \\ 0 & 4 & 6 \end{pmatrix} \begin{pmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{pmatrix} = \begin{pmatrix} a & 2b & c \\ a & 3b & 2c \\ a & 4b & 6c \end{pmatrix},$$

$$EB = \begin{pmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{pmatrix} \begin{pmatrix} 1 & 2 & 1 \\ 1 & 3 & 2 \\ 0 & 4 & 6 \end{pmatrix} = \begin{pmatrix} a & 2a & a \\ b & 3b & 2b \\ c & 4c & 6c \end{pmatrix}.$$

In general, AE has:
1st col. $= a\times$ 1st col. of A,
2nd col. $= a\times$ 2nd col. of A,
3rd col. $= a\times$ 3rd col. of A,
EA has:
1st row $= a\times$ 1st row of A,
2nd row $= a\times$ 2nd row of A,
3rd row $= a\times$ 3rd row of A.

NHB