livprob5.tex

PROBLEMS 5. 24.2.2014

The Bivariate Normal Distribution (continued from Problems 4). Show that:

Q1. $var[Y|X] = \sigma_2^2(1-\rho^2)$.

Q2. The correlation coefficient of X, Y is ρ.

Q83 The density f has elliptical contours [i.e., the curves $f(x,y)$ constant are ellipses].

Q4. The joint MGF and joint CF of X, Y are

$$M_{X,Y}(t_1,t_2) = M(t_1,t_2) := E[\exp\{t_1 X + t_2 Y\}]$$

$$= \exp(\mu_1 t_1 + \mu_2 t_2 + \frac{1}{2}[\sigma_1^2 t_1^2 + 2\rho\sigma_1\sigma_2 t_1 t_2 + \sigma_2^2 t_2^2]),$$

$$\phi_{X,Y}(t_1,t_2) = \phi(t_1,t_2) := E[\exp\{it_1 X + it_2 Y\}]$$

$$= \exp(i\mu_1 t_1 + i\mu_2 t_2 - \frac{1}{2}[\sigma_1^2 t_1^2 + 2\rho\sigma_1\sigma_2 t_1 t_2 + \sigma_2 t_2^2]).$$

Q5. X, Y are independent if and only if $\rho = 0$.

Note. For those of you with a background in Statistics, this will be familiar material. It is included here as it serves as a very concrete illustration of the more abstract conditioning of Week 4 via the Radon-Nikodym Theorem. For those of you without a background in Statistics: the key here is *completing the square* (the method you first encountered in learning how to solve quadratic equations). If you need help, find a good textbook on Statistics and look up 'bivariate normal distribution' in the index. One possibility is
[BF] N. H. BINGHAM & John M. FRY, *Regression: Linear models in statistics*, Springer Undergrad. Math. Series (SUMS), 2010, §1.5.

NHB