liv0.tex

MATH482 MATHEMATICAL FINANCE

N. H. BINGHAM

Liverpool University, Monday 2 February – Monday 11 May 2015

Course text: Ch. 1-6 of
[BK] N. H. BINGHAM and Rüdiger KIESEL: *Risk-neutral valuation: Pricing and hedging of financial derivatives*, 2nd ed., CUP, 2004.
Course Website: Math482 link on my Imperial College homepage (Imperial College > Mathematics Department > Staff > Staff List > Bingham > Homepage).

CONTENTS

III. STOCHASTIC PROCESSES IN DISCRETE TIME [1 week: W4,5].
§1. Filtrations and information flow.
§2. Discrete-parameter stochastic processes.
§3. Discrete-parameter martingales.
§4. Martingale convergence.
§5. Martingale transforms.
§6. Stopping times and optional stopping.
§7. The Snell envelope and optimal stopping.
§8. Doob decomposition.
§9. Examples.

IV. MATHEMATICAL FINANCE IN DISCRETE TIME [$2\frac{1}{2}$ weeks: W5-7].
§1. The model.
§2. Viability: existence of equivalent martingale measures (EMMs).
§3. Complete markets: uniqueness of equivalent martingale measures.
§4. The Fundamental Theorem of Asset Pricing: Risk-Neutral Valuation.
§5. European options. The discrete Black-Scholes formula.
§6. Continuous-time limit of the binomial model.
§7. More on European options.
§8. American options.

V. STOCHASTIC PROCESSES IN CONTINUOUS TIME [$1\frac{1}{2}$ weeks: W7-9].
§1. Filtrations; finite-dimensional distributions.
§2. Classes of processes.
§3. Brownian motion.
§4. Quadratic variation of Brownian motion.
§5. Stochastic integrals; Itô calculus.
§6. Stochastic differential equations; Itô's Lemma.

VI. MATH. FINANCE IN CONTINUOUS TIME [$2\frac{1}{2}$ weeks: W9-11].
§1. Geometric Brownian motion and asset prices.
§2. The Black-Scholes model and the Black-Scholes PDE.
§3. The Feynman-Kac formula, Risk-Neutral Valuation and the (continuous) Black-Scholes formula.
§4. Girsanov's theorem and change of measure.

§5. Infinite time-horizon; Real Options (Investment Options)
§6. Further results.
Postscript

Division of Time
Week 1: I, §1-5 [2 Feb]
Week 2: I, §5-9; Prelude to Ch. II [9 Feb]
Week 3: II, §1-5 [16 Feb]
Week 4: II, §5-6; III, 1-6 [23 Feb]
Week 5: III, §6-9; IV, 1-2 [2 March]
Week 6: IV, §3-6 [9 March]
Week 7: IV, §7-8; V, 1-2 [16 March]
Week 8: V, §2-5 [23 March]
Easter break
Week 9: V, §5-6; VI, 1-2 [20 April]
Week 10: VI, §2-5 [27 April]
[May 5: Bank Holiday – No lectures]
Week 11: VI, §5-6; Postscript [11 May]
Revision Week [18 May]

Format and Examination. Lectures are 4 hours per week on Mondays 11-1 and 3-5pm. There will be 11 handouts, of 12 pages each. I aim to cover about 4 pages per hour, and have time for questions, say 4-5pm. There will be Problems each week, with Solutions the week after. The syllabus is unchanged; the course (and exam – standard format) will follow last year fairly closely (see the website for Exam and Solutions 2013, 2014, and Mock Exam with Solutions).

The Big Picture

Please bear three things together in mind throughout this course:
1. Anything important enough becomes political (M. Maurice Couve de Murville). This stuff is certainly important.
2. Politics in not an exact science (Bismark). But,
3. Mathematics is an exact science.
We will be doing lots of mathematics – in particular, we derive the Black-Scholes formula. We will extend calculus, the most powerful single weapon we have, to become probabilistic (Itô calculus) and apply it to these problems. But, there are limits to which finance, economics, or anything involving

human psychology, is mathematicisable. As always in Applied Mathematics, we have to be on guard: if we don't simplify enough, we can't do anything; if we over-simplify, we can do things, but can't trust our conclusions.

Just as important as the technical mathematics, you need to think about the systemic faults at the geofinancial/economic/political level thrown up by the crisis of 2007 on (Credit Crunch, etc.). Any prospective employer in the financial services industry should ask you questions about this, and your views on it, in interview. Some of my views are in
N. H. BINGHAM: The Crash of 2008: A mathematician's view. *Significance* **5** no. 4 (2008), 173-175.
For some of my further views, see my homepage, Talks > Crash.

Background and general interest
[B1] Peter L. BERNSTEIN: *Capital ideas: The improbable origins of modern Wall Street*. New York: The Free Press, 1992.
[B2] Peter L. BERNSTEIN: *Against the Gods: The remarkable story of risk*. Wiley, 1996.
[S] Robert L. SHILLER: *Irrational exuberance*. Princeton Univ. Press, 2000.
[G] Alan GREENSPAN, *The age of turbulence*. Penguin, 2007.

I thoroughly recommend [G] – but get the latest edition of it that you can. The author was Chairman of the US Federal Reserve (Fed) 1987-2006. His views up to 2007 were largely Panglossian optimism (markets know best, and are self-correcting, etc.). The ongoing problems since have forced a re-think; see the epilogues to later editions, his evidence to the House Committee, etc.

Books for reference.
[BF] N. H. BINGHAM and J. M. FRY: *Regression: Linear models in statistics*. Springer Undergraduate Mathematics Series (SUMS), Springer, 2010 (regression is about *conditioning* – informally, using what we know).
[CR] John C. COX and Mark RUBINSTEIN: *Options markets*. Prentice Hall, 1985.
[H1] HULL, J. (1995): *Introduction to futures and options markets* (2nd ed), Prentice-Hall, ('baby Hull'), or
[H2] HULL, J. (1993): *Options, futures and other derivative securities* (2nd ed.), Prentice-Hall ('Hull').
[E] Alison ETHERIDGE: *A course in financial calculus*, CUP, 2002 [a classroom revision of [BR] below];
[BR] Martin BAXTER and Andrew RENNIE: *Financial calculus: an intro-*

duction to derivatives pricing, CUP, 1996.
[DP] Avinash K. DIXIT and Robert S. PINDYCK: *Investment under uncertainty*. Princeton University press, 1994.
[PS] G. PESKIR and A. N. SHIRYAEV: *Optimal stopping and free-boundary problems*. Birkhäuser, 2006.
[WHD] WILMOTT, P., HOWISON, S. & DEWYNNE, J. (1995): *The mathematics of financial derivatives: A student introduction*, Oxford Financial Press ['PDE with everything'];

Mathematics, for reference
[D] J. L. DOOB: *Stochastic processes*, Wiley, 1953.
[N] J. NEVEU: *Discrete-parameter martingales*, North-Holland, 1975.
[KS] KARATZAS, I. & SHREVE, S. (1988): *Brownian motion and stochastic calculus*. Graduate Texts in Math. **113**, Springer.
[RY] REVUZ, D. & YOR, M. (1999): *Continuous martingales and Brownian motion*. Grundlehren der math. Wiss. **293**, Springer, 3rd ed. (1st ed. 1991, 2nd ed. 1994,).
[RW1] ROGERS, L. C. G. & WILLIAMS, D. (1994): *Diffusions, Markov processes and martingales, Volume 1: Foundation*, 2nd ed., Wiley (1st ed. 1970).
[RW2] ROGERS, L. C. G. & WILLIAMS, D. (1987): *Diffusions, Markov processes and martingales, Volume 2: Itô calculus*. Wiley.