MATH482 EXAMINATION, 2013

Six questions, do four; 25 marks each

Q1. (a) Discuss the types of risk facing financial institutions, including: market risk; credit risk; operational risk; liquidity risk; model risk.
(b) Comment briefly on stress testing by financial regulators.

Q2. (a) Define volatility.
(b) Comment briefly on: historic volatility; implied volatility; the volatility surface.
(c) How do option prices depend on volatility, and why?
(d) Discuss the effect of trading volume on volatility, and its implications for market stability.

Q3. (a) Explain briefly, without proofs, the discrete Black-Scholes formula, the continuous Black-Scholes formula, and the relationship between them.
(b) Neither formula involves the mean return rate μ on the stock: why not?
(c) Describe briefly how to find the value and the continuation region of an American put option.

Q4. (a) For a filtration $(\mathcal{F}_n)$, what is meant by saying that $C = (C_n)$ is previsible (or predictable)?
(b) For an $(\mathcal{F}_n)$-martingale $X = (X_n)$, define the martingale transform $C \bullet X$ of X by C.
(c) Show that if C is bounded and previsible, $C \bullet X$ is a martingale null at 0.
(d) What is the interpretation of $C \bullet X$ in terms of trading?
(e) What happens in continuous time?

Q5. (a) Give the stochastic differential equation of geometric Brownian motion, and its interpretation in terms of the stock-price dynamics of the Black-Scholes model.
(b) Show how this stochastic differential equation changes when we discount by the riskless interest rate r.
(c) Show how it changes further when we apply Girsanov's theorem to change to the equivalent martingale (or risk-neutral) measure.
(d) What is the role of the representation theorem for Brownian martingales

here?

Q6. (a) Formulate the problem of real options as an optimal-stopping problem.
(b) Show that we may restrict to the case $0 < \mu < r$, where μ is the mean return on the investment and r is the riskless interest rate.
(c) Obtain the fundamental quadratic equation, with roots $p_2 < 0$, $1 < p_1$.
(d) Show that one should not invest the necessary capital I unless the initial value is at least qI, where $q := p_1/(p_1 - 1) > 1$.

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