MATH482 MOCK EXAMINATION, 2013

Six questions, do four; 25 marks each

Q1. Discuss briefly:
(i) arbitrage;
(ii) completeness of markets;
(iii) equivalent martingale measures;
(iv) risk-neutral valuation.

Q2. We assume throughout that our markets are perfect, or frictionless. This entails a number of assumptions: mention the principal ones, and discuss briefly.

Q3. State and prove the put-call parity relationship between prices of call and put options.

Q4. Show that American and European call options are equivalent: i.e., that it is never optimal to exercise an American call early.

Q5. (i) State the Ornstein-Uhlenbeck stochastic differential equation for a process $V = (V_t)$, and interpret the terms that appear in it.
(ii) Solve this equation.
(iii) Obtain the limit distribution of V_t as $t \to \infty$.
(iv) Obtain the covariance function, and find its limit as $t \to \infty$.
(v) Show that V is Markov.
(vi) Explain what is meant by saying that V is mean-reverting, and the relevance of this model to interest-rate theory.

Q6. Show how to formulate an American put option with an infinite time-horizon as an optimal-stopping problem.

Show further how to formulate this as a free-boundary problem.

Obtain the fundamental quadratic. Hence or otherwise, solve the problem.

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