

PROBLEMS 8. 23.3.2015

Q1. The *moment-generating function* $M_X(t)$ of a random variable X is defined by $M_X(t) := E[e^{tX}]$. If X has the normal distribution $N(\mu, \sigma^2)$, with density

$$f(x) := \frac{1}{\sqrt{2\pi}\sigma} \exp\left\{-\frac{1}{2}(x - \mu)^2/\sigma^2\right\},$$

show (by completing the square, or otherwise) that

$$M_X(t) = \exp\left\{\mu t + \frac{1}{2}\sigma^2 t^2\right\}.$$

Q2. The *lognormal distribution* $\log N(\mu, \sigma^2)$ is defined as the distribution of $X := e^Y$, where Y is $N(\mu, \sigma^2)$.

(i) By using the result of Question 1, or otherwise, show that X has mean

$$E[X] = \exp\left\{\mu + \frac{1}{2}\sigma^2\right\}.$$

(ii) Explain without proof why the prices of stocks in the Black-Scholes model are log-normally distributed.

NHB