ltcc0.tex
Lecture 0. 3.11.2014.

London Taught Course Centre

MEASURE-THEORETIC PROBABILITY

Professor N. H. BINGHAM, Autumn 2014

Mathematics Department, Imperial College, 6M47; 020-7594 2085;
n.bingham@ic.ac.uk

Course website: My homepage, link to Measure-Theoretic Probability. This also contains past exam papers + solutions.

This 10-hour course [5 weeks, 2 hours each] can cover only the bare essentials of a vast subject, and of course there is very much more to say. For a 20-hour version see the Stochastic Analysis link on my homepage [10 x 2 hours, LSE]; for a 30-hour version see the Stochastic Processes link on my homepage [30 x 1 hour, Imperial]. For a range of useful books, see the References after the Table of Contents below.

Contents

Week 1. 3.11.2014. Chapter I: Probability backgound.
§1. Measure.
§2. Integral.
§3. Probability.
§4. Equivalent measures and the Radon-Nikodym theorem.

Week 2. 10.11.2014. Chapter II: Conditioning. Stochastic Processes.
§1. Conditional expectation.
§2. Properties of conditional expectation.
§3. Filtrations.
§4. Discrete-parameter stochastic processes.
§5. Stochastic processes in continuous time.
§6. Renewal processes; Poisson process.

Week 3. 17.11.2014. Chapter III: Martingales.
§1. Discrete-parameter martingales.
§2. Martingale convergence.
§3. Uniformly integrable martingales.
§4. Stopping times and optional stopping.
§5. Doob decomposition.
§6. Examples.
§7. Continuous-parameter martingales.
§8. Poisson processes; Lévy processes

Week 4. 24.11.2014. Chapter IV: Stochastic processes in continuous time. Brownian motion.
§1. Markov processes.
§2. Gaussian processes.
§3. Brownian motion.

Week 5. 1.12.2014. Chapter V: Itô (stochastic) calculus. Weak convergence.
§1. Quadratic variation.
§2. Itô integral.
§3. Itô's formla.
§4. Weak convergence.

REFERENCES

[A] D. B. APPLEBAUM (2004): *Lévy processes and stochastic calculus*, CUP.

[Bach] L. BACHELIER (2006): Louis Bachelier's *Theory of speculation*: The origins of modern finance. Translated and with a commentary by Mark Davis and Alison Etheridge; foreword by Paul A. Samuelson, Princeton UP.

[Ber] J. BERTOIN (1996): *Lévy processes.* Cambridge Tracts 121, CUP.

[Bil] P. BILLINGSLEY (1968): *Convergence of probability measures*, Wiley.

[BF] N. H. BINGHAM and John M. FRY: *Regression: Linear models in statistics.* Springer Undergraduate Mathematics Series (SUMS), 2010.

[BK] N. H. BINGHAM and Rüdiger KIESEL: *Risk-neutral valuation: Pricing and hedging of financial derivatives*, 2nd ed. Springer Finance, 2004 (1st ed. 1998).

[Brei] L. BREIMAN (1968), *Probability*, Addison-Wesley.

[D] J. L. DOOB (1953), *Stochastic processes*, Wiley.

[F1] W. FELLER (1968): *An introduction to probability theory and its applications*, Vol. 1, 3rd ed., Wiley (1st ed. 1950, 2nd ed. 1957).

[F2] W. FELLER (1971): *An introduction to probability theory and its applications*, Vol. 2, 2nd ed., Wiley (1st ed. 1966).

[GS] G. R. GRIMMETT and D. R. STIRZAKER (2001), *Probability and random processes*, 3rd ed., OUP (1st ed. 1982, 2nd ed. 1992).

[Joh] I. M. JOHNSTONE (2011+): *Function estimation and Gaussian sequence models*, http://www-stat.stanford.edu/ imj

[Kal] O. KALLENBERG (2002): *Foundations of modern probability*, 2nd ed., Springer.

[K-S] I. KARATZAS and S. E. SHREVE (1988): *Brownian motion and stochastic calculus*, Springer.

[Kin] J. F. C. KINGMAN (1993): *Poisson processes*, OUP.

[Kol] A. N. KOLMOGOROV (1933): *Grundbegriffe der Wahrscheinlichkeitsrechnung*, Springer.

[L-Q] T. J. LYONS and Z. QIAN (2002): *System control and rough paths*, OUP.

[Mey66] P.-A. MEYER (1966): *Probability and potentials*, Blaisdell.

[Mey76] P.-A. MEYER (1976): Un cours sur les intégrales stochastiques, *Sém. Probab. X*, *Lecture Notes in Math.* **511**, 245-400.

[Mik] T. MIKOSCH (1998): *Elementary stochastic caclulus, with finance in view*, World Scientific.

[Nev] J. NEVEU (1975): *Discrete-parameter martingales*, North-Holland.

[Ø] B. ØKSENDAL (1998): *Stochastic differential equations. An introduction with applications*, 5th ed., Springer.

[Pro] P. PROTTER (1990): *Stochastic integration and differential equations. A new approach*, Springer.

[R-Y] D. REVUZ and M. YOR (1999): *Continuous martingales and Brownian motion*, 3rd ed., Springer.

[R-W1] L. C. G. ROGERS and D. WILLIAMS (1994): *Diffusions, Markov processes and martingales, Volume 1: Foundations*, 2nd ed, Wiley.

[R-W2] L. C. G. ROGERS and D. WILLIAMS (1987): *Diffusions, Markov processes and martingales, Volume 2: Itô calculus*, Wiley.

[Ste] J. M. STEELE (2001): *Stochastic calculus and financial applications*, Springer.

[Wil91] D. WILLIAMS (1991): *Probability with martingales*, CUP.

[Wil01] D. WILLIAMS (2001): *Weighing the odds. A course in probability and statistics*, CUP.