## LTCC, MEASURE-THEORETIC PROBABILITY: EXAMINATION 2018

Q1. Given an infinite sequence  $U_n$  of independent random variables, each with the uniform distribution U[0, 1] on the unit interval (e.g., from an 'ideal random number generator'), show how to use the  $U_n$  to simulate

(a) an infinite sequence  $Z_n$  of independent standard normal random variables;

(b) a Brownian motion B = (B(t)).

Q2. By using the Cantor diagonalisation process, or otherwise, show further how, given a single U with distribution U[0, 1], to construct

(a) an infinite sequence of independent  $U_n$  as above;

(b) an infinite sequence  $(B_n)$  of independent Brownian motions.

Q3. How does one construct independent Brownian bridges instead of Brownian motions?

Q4. In practice, how does one adapt to life, computing power and storage, and the resolution of computer graphics, all being only finite?

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