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LONDON TAUGHT COURSE CENTRE: MOCK EXAMINATION, 2008
MEASURE-THEORETIC PROBABILITY

Q1. A.

(i) The *generating function* of a random variable X with non-negative integer values is $P(s)$, or $P_X(s)$, $:= \sum_{n=0}^{\infty} P(X = n)s^n$. Show that if X, Y are independent with generating functions P_X, P_Y, then $X + Y$ has generating function $P_{X+Y}(s) = P_X(s).P_Y(s)$.

(ii) A random variable X has the *Poisson* distribution with *parameter* λ, $X \sim P(\lambda)$, if $P(X = n) = e^{-\lambda}\lambda^n/n!$, $n = 0, 1, 2, \ldots$. Show that $P_X(s) = e^{-\lambda(1-s)}$.

(iii) If X, Y are independent, Poisson with parameters λ, μ, show that $X + Y$ is Poisson with parameter $\lambda + \mu$.

(iv) Show that $EX = \lambda$. Hence, given that $X + Y$ is Poisson, find its parameter without calculation.

B.

(i) Define a *Poisson point process* with *parameter* λ, $Ppp(\lambda)$.

(ii) If $X = (X_t)$, $Y = (Y_t)$ are independent Poisson point processes with parameters λ, μ, show that $X + Y$ is a $Ppp(\lambda + \mu)$.

(iii) Given that $X + Y$ is a Ppp, find its parameter without calculation.

Q2. Let $s_1, \sigma_2 > 0$, and let B_1, B_2 be independent standard Brownian motions. Write

$$X_t := (\sigma_1 B_1(t) + \sigma_2 B_2(t))/(\sigma_1^2 + \sigma_2^2)^{1/2}.$$

Find the mean and covariance of X. Deduce that X is standard Brownian motion.

NHB