London Taught Course Centre: Measure-Theoretic Probability

Examination, 2010

Q1. (i) For N Poisson distributed with parameter λ and $X_1, X_2, \ldots$ independent of each other and of N, each with distribution F with mean μ, variance σ^2 and characteristic function $\phi(t)$, show that the compound Poisson distribution of

$$Y := X_1 + \ldots + X_N$$

has characteristic function $\psi(t) = \exp\{-\lambda(1 - \phi(t))\}$, mean $\lambda\mu$ and variance $\lambda E[X^2]$.
(ii) Obtain the mean and variance of Y also from the Conditional Mean Formula and the Conditional Variance Formula.

Q2. (i) For $B = (B_t) = (B(t))$ standard Brownian motion, define

$$X_t := tB(1/t) \qquad (t \neq 0).$$

Show that $X = (X_t)$ is again standard Brownian motion.
(ii) Hence or otherwise, show that

$$B_t/t \to 0 \qquad a.s. \qquad (t \to \infty).$$

N. H. Bingham