

**PROBLEMS 8 29.11.2012**

Q1. Prove the *Conditional Mean Formula*: for  $\mathcal{B}$  any  $\sigma$ -field,

$$E[E[X|\mathcal{B}]] = E[X].$$

Q2. Prove the *Conditional Variance Formula*

$$\text{var}(Y) = E[\text{var}(Y|X)] + \text{var}(E[Y|X]).$$

Q3. (i) For  $N$  Poisson distributed with parameter  $\lambda$  and  $X_1, X_2, \dots$  independent of each other and of  $N$ , each with distribution  $F$  with mean  $\mu$ , variance  $\sigma^2$  and characteristic function  $\phi(t)$ , show that the compound Poisson distribution of

$$Y := X_1 + \dots + X_N$$

has characteristic function  $\psi(t) = \exp\{-\lambda(1 - \phi(t))\}$ , mean  $\lambda\mu$  and variance  $\lambda E[X^2]$ .

(ii) Obtain the mean and variance of  $Y$  also from the Conditional Mean Formula and the Conditional Variance Formula.