pfsprob35tex

PROBLEMS 5. 7.10.2013

Q1. (i) Show that the standard normal distribution $N(0,1)$ has CF $e^{-t^2/2}$.
(ii) Deduce that the general normal distribution $N(\mu,\sigma)$ has CF
$\exp\{i\mu t - \sigma^2 t^2/2\}$.

Q2. (i) Show that the symmetric exponential distribution SE with density

$$f(x) := e^{-|x|}/2$$

has CF

$$\phi(t) = 1/(1+t^2).$$

(One can do this by Real Analysis – integrate by parts twice.)
(ii) Show that the Cauchy distribution with density

$$f(x) = \frac{1}{\pi(1+x^2)}$$

has CF

$$\phi(t) = e^{-|t|}.$$

(This uses Complex Analysis, and Jordan's Lemma.)

Q3. Comment on the similarity between density and CF in Q1, and between
Q2 (i) and (ii).

Q4. (i) Show that if both AB and BA are defined, $trace(AB) = trace(BA)$.
(ii) Show that P, $I - P$ have ranks p, $n - p$.

NHB

1