pfsprob8.tex

PROBLEMS 8 28.11.2013

Q1. Prove the *Conditional Mean Formula*: for $\mathcal{B}$ any σ-field,

$$E[E[X|\mathcal{B}]] = E[X].$$

Q2. Prove the *Conditional Variance Formula*

$$var(Y) = E[var(Y|X)] + var(E[Y|X]).$$

Q3. (i) For N Poisson distributed with parameter λ and $X_1, X_2, \ldots$ independent of each other and of N, each with distribution F with mean μ, variance σ^2 and characteristic function $\phi(t)$, show that the compound Poisson distribution of

$$Y := X_1 + \ldots + X_N$$

has characteristic function $\psi(t) = \exp\{-\lambda(1 - \phi(t))\}$, mean $\lambda\mu$ and variance $\lambda E[X^2]$.
(ii) Obtain the mean and variance of Y also from the Conditional Mean Formula and the Conditional Variance Formula.

NHB