PROBABILITY FOR STATISTICS: RESIT EXAMINATION 2014

Q1. Define the *Chi-square distribution* with n *degrees of freedom*, $\chi^2(n)$. Show that it has
(i) characteristic function $1/(1-2it)^{\frac{1}{2}n}$;
(ii) density

$$f(x) = \frac{1}{2^{\frac{1}{2}n}\Gamma(\frac{1}{2}n)} x^{\frac{1}{2}n-1} \exp(-\frac{1}{2}x) \qquad (x > 0);$$

(iii) mean n, variance $2n$.

Q2. Define convergence in probability and convergence in distribution. How are these related?

State and prove the Weak Law of Large Numbers. Any statements you need (e.g., properties of characteristic functions) may be quoted without proof, but should be clearly stated.

Q3. A *compound Poisson* process of rate λ and jump-distribution F is one of the form $S = (S(t))$, $S(t) := X_1 + \cdots + X_{N(t)}$, with $N = (N(t))$ a Poisson process of rate λ and (X_n) with distribution F, independent of each other and of N.

If F has characteristic function ϕ and has finite mean and variance, find the characteristic function, mean and variance of $S(t)$.

Q4. In a Markov chain, define the terms transient, recurrent (or persistent), null, positive and mean recurrence time.

Show that in a finite Markov chain
(i) not all states can be transient;
(ii) there are no null states.

Give examples of Markov chains with all states (a) transient, (b) positive, (c) null.

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