PROBABILITY FOR STATISTICS

Professor N. H. BINGHAM

6M47; 4-2085; Office hour: Th 4-5
Autumn Term, 2014; 13 Oct – 16 Dec, Room 140
Mon 12-1, Tue 10-12, including Problems class

Contents

5. Continuous time [L22]

VI. Stochastic processes [$3\frac{1}{2}$ hours]
1. Filtrations; finite-dimensional distributions; conditional expectations [L23-24]
2. Martingales: discrete time [L24]
3. Martingales: continuous time [L25]
4. Other classes of process: [L25-26]
   Gaussian; Markov; diffusions

VII. Lévy processes [$2\frac{1}{2}$ hours]
1. Brownian motion [L26-27]
2. Poisson process; compound Poisson processes [L27]
3. Lévy-Itô decomposition [L27-28]

VIII. Extreme-value theory [1 hour] [L29]

Lecture hours

We have 30 lectures: 3 a week for Weeks 2-11.

Problems

I shall set 10 Problems. Half an hour a week (Tuesday, after the break) will be devoted to a Problems Class – going over the Solutions to that week's Problems, and answering questions.

Examination

This will be in standard format [2 hours, 4 questions]. For the previous two years' Exam + Solutions (and Mock Examination + Solutions), see the webpage.

Recommended texts

The course

Good elementary books include:
[H] John HAIGH, *Probability models*, Springer Undergraduate Mathematics Series (SUMS), Springer, 2002.
[Ch] Kai-Lai CHUNG, *Elementary probability theory with stochastic processes*, Springer, 3rd ed. 1979 (1st ed. 1974, 2nd ed. 1975; the 4th ed., 2003, with Farid Ait-Sahlia, also includes an introduction to mathematical finance).

The 'default option' for an undergraduate probability book in the UK is [GS] Geoffrey GRIMMETT and David STIRZAKER, *Probability and ran-*

dom processes, OUP, 3rd ed., 2001, 596p (2nd ed. 1992; don't use the 1st ed., 1982; 3rd ed. Section 13.10 contains a brief introduction to option pricing in mathematical finance and the Black-Scholes formula).
There is far more material here than we can cover.

Measure Theory

You may be aware that Measure Theory is the mathematics of length, area, volume etc. (Lebesgue measure), and that this extends generally. A measure of total mass 1 is called a *probability measure*. There is a corresponding integration theory. In the Lebesgue case, we get the *Lebesgue integral*, which extends the Riemann integral of undergraduate mathematics (the 'Sixth Form integral', formalised with epsilons etc.), is much better and easier to handle, but is harder to set up. For a probability measure, the integral is called the *expectation*; you may have met elementary cases of this already. A proper treatment needs Measure Theory, but we will make no attempt to follow such an approach here ([GS] occasionally does so). For background here (and you will definitely need Measure Theory if you take the subject further!), see e.g.

[P] David POLLARD, *A user's guide to measure-theoretic probability*, CUP, 2002;

[S] René L. SCHILLING, *Measures, integrals and martingales*, CUP, 2005.

Links

You may find the Stochastic Processes link on my homepage useful here. I will refer to Lecture n there as 'SP, Ln'. I shall also refer to the Introductory Lectures on Statistics (on the SP link, at the beginning – page n there is 'IS, n'), and to the links on my homepage to Stochastic Analysis (SA) and Statistical Methods for Finance (SMF).

Statistics

Some of what we do may be found in

[BF] N. H. BINGHAM and John M. FRY: *Regression: Linear models in statistics*. SUMS, Springer, 2010.

Reference books

We make occasional reference to (excellent) books that are more advanced, e.g.

[K] Olav KALLENBERG, *Foundations of modern probability*, 2nd ed., Springer, 2002 [1st ed. 1997].

[Bog] V. I. BOGACHEV, *Measure theory*, Volumes 1, 2, Springer, 2007;

[Dud] R. M. DUDLEY, *Real analysis and probability*, 2nd ed., 2002, Cambridge UP [1st ed., Wadsworth, 1989].

[D] J. L. DOOB, *Stochastic processes*, Wiley, 1953.
[vdVW] A. W. van der VAART & J. A. WELLNER, *Weak convergence and empirical processes, with applications to statistics*, Springer, 1996, Ch. 2.
[Bil] P. BILLINGSLEY, *Convergence of probability measures*, Wiley, 1968 [2nd ed. 1999).
[GnK] B. V. GNEDENKO and A. N. KOLMOGOROV, *Limit theorems for sums of independent random variables*, Addison-Wesley, 1954.

We also cite occasionally:

Statistics
[C] H. CRAMÉR, *Mathematical methods of statistics*, Princeton University Press, 1946.
[R] C. R. RAO, *Linear statistical inference and its applications*, 2nd ed., Wiley, 1973 [1st ed. 1965].
[KS1] M. G. KENDALL and A. STUART, *The advanced theory of statistics. Volume 1: Distribution theory*, 4th ed., Griffin, 1977.
Probability
[F] W. FELLER, *An introduction to probability theory and its applications*, Volume 1, 3rd ed., Wiley, 1968.
[N] J. R. NORRIS, *Markov chains*, CUP, 1998.
[M] P. A. P. MORAN: *An introduction to probability theory*, OUP, 1968.
[CM] D. R. COX & H. D. MILLER: *The theory of stochastic processes*, Chapman & Hall, 1965.
[E] W. J. EWENS: *Mathematical population genetics*, Springer, 1979.
[Hag] O. HÄGGSTRÖM: *Finite Markov chains and algorithmic applications.* LMS Student Texts **52**, CUP, 2002.
[MT] S. MEYN and R. L. TWEEDIE, *Markov chains and stochastic stability*, 2nd ed., CUP, 2009.
Linear Algebra
[Hal] P. R. HALMOS: *Finite-dimensional vector spaces.* Undergraduate Texts in Mathematics, Springer, 1974 (Van Nostrand, 1942, 1958).
[HJ] R. A. HORN and C. R. JOHNSON: *Matrix analysis*, CUP, 1985.
[Sen] E. SENETA: *Non-negative matrices and Markov chains*, 2nd ed. Springer, 2006 (1st ed. 1973, 1981).
Extreme-value theory
[Col] Stuart COLES: *An introduction to statistical modelling of extreme values.* Springer, 2001.