pfsprob2.tex

PROBLEMS 2. 21.10.2014

Q1. (i) Show that if $x = (x_1, \ldots, x_n)$ is a (column) vector with the x_i independent and identically distributed (iid) standard normal ($N(0,1)$), and

$$y := Ox$$

with O an orthogonal matrix, then the components y_i of y are again iid $N(0,1)$.
(ii) Show how to construct a *Helmert matrix* – an orthogonal matrix with first row

$$e_1 = (1/\sqrt{n}, \ldots, 1/\sqrt{n}).$$

(iii) For a normal population $N(\mu, \sigma^2)$, show that the sample mean $\bar{X}$ and the sample variance S^2 are independent.

Q2. Show that the chi-square distribution $\chi^2(n)$ with n degrees of freedom has
(i) mean n and variance $2n$,
(ii) MGF $M(t) = 1/(1-2t)^{\frac{1}{2}n}$ for $t < \frac{1}{2}$,
(iii) density

$$f(x) = \frac{1}{2^{\frac{1}{2}n}\Gamma(\frac{1}{2}n)}.x^{\frac{1}{2}n-1}\exp(-\frac{1}{2}x) \qquad (x > 0).$$

NHB