pfsprob3.tex

## PROBLEMS 3. 28.10.2014

Q1. Show that Stirling's formula gives

$$\Gamma(x+a) \sim x^a \Gamma(x) \qquad (x \to \infty).$$

Q2. You may quote that the *Student* t statistic

$$t := \sqrt{n-1}(\bar{X}-\mu)/S$$

has distribution t(n-1), where t(r) has density

$$f(x) = \frac{\Gamma(\frac{1}{2}r + \frac{1}{2})}{\sqrt{\pi r}\Gamma(\frac{1}{2}r)} \left(1 + \frac{x^2}{r}\right)^{-\frac{1}{2}(r+1)}.$$

Show that as  $n \to \infty$ 

$$t(n) \to \Phi = N(0,1)$$

(i) by using Q1 and the density above;

(ii) by using the Law of Large Numbers and the definition of the Student *t*-statistic.

Q3. By considering the quadratic

$$Q(\lambda) := \frac{1}{n} \sum_{i=1}^{n} (\lambda(x_i - \bar{x}) + (y_i - \bar{y}))^2,$$

show that the sample correlation coefficient r satisfies (i)  $-1 \leq r \leq 1$ 

(ii)  $r = \pm 1$  iff there is a linear relationship between  $x_i$  and  $y_i$ ,

$$ax_i + by_i = c \quad (i = 1...n).$$

Q4. By considering the quadratic

$$Q(\lambda) := E[(\lambda(x - \bar{x}) + (y - \bar{y}))^2],$$

show that the population correlation coefficient  $\rho$  satisfies (i)  $-1 \le \rho \le 1$ ,

(ii)  $\rho = \pm 1$  iff there is a linear relationship between x and y, ax + by = c with probability 1.

NHB