pfsprob10.tex

PROBLEMS 10 15.12.2015 (Prob + Soln 10 with Soln 9, last lecture, L28: two lectures lost, Commemoration Day)

Q1. Prove the Conditional Mean Formula: for \mathcal{B} any σ -field,

$$E[E[X|\mathcal{B}]] = E[X].$$

Q2. Prove the Conditional Variance Formula

$$var(Y) = E[var(Y|X)] + var(E[Y|X]).$$

Q3. (i) For N Poisson distributed with parameter λ and X_1, X_2, \ldots independent of each other and of N, each with distribution F with mean μ , variance σ^2 and characteristic function $\phi(t)$, show that the compound Poisson distribution of

$$Y := X_1 + \ldots + X_N$$

has characteristic function $\psi(t) = \exp\{-\lambda(1-\phi(t))\}\)$, mean $\lambda\mu$ and variance $\lambda E[X^2]$.

(ii) Obtain the mean and variance of Y also from the Conditional Mean Formula and the Conditional Variance Formula.

NHB