pfsprob3.tex

PROBLEMS 3. 2.11.2015

Q1. Show that Stirling's formula gives

$$\Gamma(x+a) \sim x^a \Gamma(x) \qquad (x \to \infty).$$

Q2. You may quote that the *Student* t statistic

$$t := \sqrt{n-1}(\bar{X} - \mu)/S$$

has distribution $t(n-1)$, where $t(r)$ has density

$$f(x) = \frac{\Gamma(\frac{1}{2}r + \frac{1}{2})}{\sqrt{\pi r}\,\Gamma(\frac{1}{2}r)} \Big(1 + \frac{x^2}{r}\Big)^{-\frac{1}{2}(r+1)}.$$

Show that as $n \to \infty$

$$t(n) \to \Phi = N(0,1)$$

(i) by using Q1 and the density above;
(ii) by using the Law of Large Numbers and the definition of the Student t-statistic.

Q3. By considering the quadratic

$$Q(\lambda) := \frac{1}{n} \sum_{i=1}^{n} (\lambda(x_i - \bar{x}) + (y_i - \bar{y}))^2,$$

show that the sample correlation coefficient r satisfies
(i) $-1 \le r \le 1$
(ii) $r = \pm 1$ iff there is a linear relationship between x_i and y_i,

$$a x_i + b y_i = c \quad (i = 1 \ldots n).$$

Q4. By considering the quadratic

$$Q(\lambda) := E[(\lambda(x - \bar{x}) + (y - \bar{y}))^2],$$

show that the population correlation coefficient ρ satisfies
(i) $-1 \le \rho \le 1$,
(ii) $\rho = \pm 1$ iff there is a linear relationship between x and y, $ax + by = c$ with probability 1.

NHB