

**PROBLEMS 5. 16.11.2015**

Q1. (i) Show that the standard normal distribution  $N(0, 1)$  has CF  $e^{-t^2/2}$ .  
(ii) Deduce that the general normal distribution  $N(\mu, \sigma)$  has CF  $\exp\{i\mu t - \sigma^2 t^2/2\}$ .

Q2. (i) Show that the symmetric exponential distribution  $SE$  with density

$$f(x) := e^{-|x|}/2$$

has CF

$$\phi(t) = 1/(1 + t^2).$$

(One can do this by Real Analysis – integrate by parts twice.)

(ii) Show that the Cauchy distribution with density

$$f(x) = \frac{1}{\pi(1 + x^2)}$$

has CF

$$\phi(t) = e^{-|t|}.$$

(This uses Complex Analysis, and Jordan's Lemma.)

Q3. Comment on the similarity between density and CF in Q1, and between Q2 (i) and (ii).

Q4. (i) Show that if both  $AB$  and  $BA$  are defined,  $\text{trace}(AB) = \text{trace}(BA)$ .  
(ii) Show that  $P$ ,  $I - P$  have ranks  $p$ ,  $n - p$ .

NHB