smfprob2.tex

SMF PROBLEMS 2. 28.1.2011

Q1. In the regression model

$$y = A\beta + \epsilon$$

(data y an n-vector, the design matrix A an $n \times p$ matrix of constants, β a p-vector of parameters, ϵ an n-vector of errors with independent $N(0, \sigma^2)$ components), show that the maximum-likelihood estimators, and also the least-squares estimators, are

$$\hat{\beta} = (A^T A)^{-1} A^T y.$$

Show also that (in the notation of lectures and Problems 1)

$$Py = A\hat{\beta}.$$

Q2. Show that the maximum-likelihood estimator of the error variance σ^2 is $\hat{\sigma}^2 = SSE/n$, where the *sum of squares for error* SSE is given by

$$SSE = (y - A\hat{\beta})^T (y - A\hat{\beta}) = y^T (I - P) y.$$

(In fact it is better to use the *unbiased* estimator of σ^2, which is $\hat{\sigma}^2 = SSE/(n-p)$; we quote this. We have p parameters β_i to estimate; for each, we 'lose a degree of freedom', which reduces the effective sample size to $n-p$.)

Q3. If $x \sim N(\mu, \Sigma)$, show that linear forms AX, BX in x are independent iff

$$A \Sigma B^T = 0.$$

Q4. If X has components X_i independent $N(0,1)$ and $Y := BX$ with B orthogonal, show that $Y =_d X$.

NHB