

**SMF PROBLEMS 3. 4.2.2011**

Q1. The  $AR(p)$  process  $(X_t)$  is given by

$$X_t = \phi_1 X_{t-1} + \cdots + \phi_p X_{t-p}, \quad (\epsilon_t) \quad WN(\sigma^2).$$

- (i) State without proof the condition for stationarity.
- (ii) Derive the Yule-Walker equations for the autocorrelation  $(\rho_k)$ .
- (iii) State the general solution of the Yule-Walker equations.

Q2. The  $MA(1)$  process  $(X_t)$  is given by

$$X_t = \epsilon_t + \theta \epsilon_{t-1}, \quad |\theta| < 1, \quad (\epsilon_t) \quad WN(\sigma^2).$$

Find

- (i) the variance  $\gamma_0 = \text{var} X_t$ ,
- (ii) the autocovariance  $\gamma_k = \text{cov}(X_t, X_{t+k})$ ,
- (iii) the autocorrelation  $\rho_k = \text{corr}(X_t, X_{t+k})$ .

Q3. The time-series model is given by

$$X_t = X_{t-1} - \frac{1}{4}X_{t-2} + \epsilon_t + \frac{1}{2}\epsilon_{t-1}, \quad (\epsilon_t) \quad WN(\sigma^2).$$

- (i) Classify  $(X_t)$  within the  $ARIMA$  class.
- (ii) Show that  $(X_t)$  is stationary and invertible.

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