smfprob10.tex

SMF PROBLEMS 10. 8.6.2012

Q1. The AR(p) process (X_t) is given by

$$X_t = \phi_1 X_{t-1} + \dots + \phi_p X_{t-p}, \qquad (\epsilon_t) \quad WN(\sigma^2).$$

(i) State without proof the condition for stationarity.

(ii) Derive the Yule-Walker equations for the autocorrelation (ρ_k) .

(iii) State the general solution of the Yule-Walker equations.

Q2. The MA(1) process (X_t) is given by

$$X_t = \epsilon_t + \theta \epsilon_{t-1}, \qquad |\theta| < 1, \qquad (\epsilon_t) \quad WN(\sigma^2).$$

Find

(i) the variance $\gamma_0 = varX_t$,

(ii) the autocovariance $\gamma_k = cov(X_t, X_{t+k}),$

(iii) the autocorrelation $\rho_k = corr(X_t, X_{t+k})$.

Q3. The time-series model is given by

$$X_t = X_{t-1} - \frac{1}{4}X_{t-2} + \epsilon_t + \frac{1}{2}\epsilon_{t-1}, \quad (\epsilon_t) \quad WN(\sigma^2).$$

(i) Classify (X_t) within the ARIMA class.

(ii) Show that (X_t) is stationary and invertible.

Q4. If X has components X_i independent N(0,1) and Y := BX with B orthogonal, show that $Y =_d X$.

NHB