smfprob4.tex

SMF PROBLEMS 4. 18.5.2012

Q1. Does the Cauchy location family have any non-trivial sufficient statistics?

Q2 Two normal means – equal (unknown) variances). The problem of testing for equality of two normal means with variances unknown and not necessarily equal is a notoriously difficult problem, the *Behrens-Fisher prolblem*.

We restrict here to the case of two normal means with variances σ^2 unknown but assumed equal. Draw independent samples $X_i \sim N(\mu_1, \sigma^2)$ of size m and $Y_j \sim N(\mu_2, \sigma^2)$ of size n. Then

$$\bar{X} \sim N(\mu_1, \sigma^2/m),\ \bar{Y} \sim N(\mu_1, \sigma^2/m),\ mS_x^2/\sigma^2 \sim \chi^2(m-1),\ mS_y^2/\sigma^2 \sim \chi^2(m-1),$$

with all four independent. So, writing $\Sigma_1 := \sum_1^m (X_i - \bar{X})^2$,
$\Sigma_2 := \sum_1^n (Y_j - \bar{Y})^2$, $(m\Sigma_1 + n\Sigma_2)/\sigma^2 \sim \chi^2(m+n-1)$,

$$\bar{X} - \bar{Y} \sim N(\mu_1 - \mu_2, \sigma^2(\frac{1}{m} + \frac{1}{n}) = N(\mu_1 - \mu_2, \sigma^2(\frac{m+n}{mn}),$$

$$t := \sqrt{m+n-2}.\sqrt{\Big(\frac{mn}{m+n}\Big)}[(\bar{X} - \bar{Y} - (\mu_1 - \mu_2)]/\sqrt{m\Sigma_1 + n\Sigma_2} \sim t(m_n - 2).$$

So the traditional test here is the *t-test*: reject $H_0: \ \mu_1 = \mu_2$ iff $|t|$ is too big.

Show that the Likelihood Ratio Test for $H_0: \ \mu_1 = \mu_2$ v. H_1: μ_i unrestricted reduces to the t-test.

Q3. We draw a sample of size n from the uniform distribution $U(0, \theta)$, where θ has prior density $\lambda e^{-\lambda\theta}$ (i.e. $\theta \sim E(\lambda)$). Find the posterior density of θ to within a multiplicative constant.

NHB