smfprob6.tex

SMF PROBLEMS 6. 25.5.2012

Q1. For the shifted exponential distribution with parameter $\theta > 0$ (density $e^{-(x-\theta)}$ for $x > \theta$),
(i) find the MLE;
(ii) find for each n a sufficient statistic;
(iii) if θ has prior density $\lambda e^{-\lambda\theta}$ ($\theta \sim E(\lambda)$), find the posterior density to within a multiplicative constant.

Q2. We quote (Edgeworth's Theorem: V.3, Day 7) that for the multivariate normal distribution $N(\mu, \Sigma)$ with mean vector μ and covariance matrix Σ, the density has the form $const.\exp\{-\frac{1}{2}(x-\mu)^T\Sigma^{-1}(x-\mu)\}$. For vectors y, u, one has

$$y|u \sim N(X\beta + Zu, R), \qquad u \sim N(0, D).$$

Show that

$$u|y \sim N(\mu, \Sigma), \quad \mu = (Z^T R^{-1} Z + D^{-1})^{-1} Z^T R^{-1}(y - X\beta), \quad \Sigma = (Z^T R^{-1} Z + D^{-1})^{-1}.$$

(The apparently complicated algebra should not suggest that this example is artificial! It is standard theory when dealing with *random effects* in regression, and originally arose in statistical studies of breeding of dairy cattle. For background and details, see e.g. [BF], 208-9.)

NHB