smfprob8.tex

SMF PROBLEMS 8. 1.6.2012

Q1. The chi-square distribution with n degrees of freedom (df), $\chi^2(n)$, is defined to be that of $X_1^2 + \ldots + X_n^2$ with the X_i independent N(0, 1). Show that $\chi^2(n)$ has

(i) mean n and variance 2n;

(ii) moment-generating function (MGF) $M(t) = 1/(1-2t)^{\frac{1}{2}n} (t < \frac{1}{2});$ (iii) density

$$f(x) = \frac{1}{2^{\frac{1}{2}n} \Gamma(\frac{1}{2}n)} . x^{\frac{1}{2}n-1} \exp\{-\frac{1}{2}x\} \quad (x > 0).$$

Q2. For A the design matrix, show that

(i) $P := A(A^T A)^{-1} A^T$ is an $n \times n$ symmetric idempotent matrix $(P^2 = P)$ – a projection;

(ii) I - P is a projection.

The trace tr(A) of a matrix A is the sum of its diagonal elements. Show that

(a) tr(A + B) = tr(A) + trB), tr(AB) = tr(BA);(b) tr(P) = p, tr(I - P) = n - p.

NHB