m2pm3prob6(11).tex

M2PM3 PROBLEMS 6. 24.5.2013

Q1. With $\chi^2(n)$ defined as the sum of $X_1^2 + \ldots + X_n^2$ with x_i iid $N(0,1)$, show that $\chi^2(n)$ has
(i) mean n and variance $2n$;
(ii) MGF $M(t) = 1/(1-2t)^{\frac{1}{2}n}$ for $t < \frac{1}{2}$;
(iii) density

$$f(x) = \frac{1}{2^{\frac{1}{2}n}\Gamma(\frac{1}{2}n)} x^{\frac{1}{2}n-1} \exp(-\frac{1}{2}x) \qquad (x > 0).$$

Q2. With A the design matrix and $P := A(A^TA)^{-1}A^T$, show that:
(i) P is a symmetric projection;
(ii) $I - P$ is a symmetric projection;
(iii) $tr(P) = p$ and $tr(I - P) = n - p$.

NHB