smfprob8(13a).tex

SMF PROBLEMS 8. 6.12.2013

Q1. (i) For a Bernoulli distribution $B(p)$ with uniform prior on $p \in [0,1]$, show that the posterior distribution is a Beta distribution, and find the parameters.
(ii) Repeat with a Beta prior $B(\alpha, \beta)$.

Q2. For the Bernoulli distribution $B(p)$, find
(i) the information per reading;
(ii) the Jeffreys prior.

Q3. Find the mean of $B(\alpha, \beta)$.

Q4. Hence find the posterior mean in Q1(ii), and interpret this as the sample size n increases.

Q5 (*Convolutions of Gammas and Euler's integral for the Beta function*). Write f_α for the exponential density with parameter α:

$$f_\alpha(x) = x^{\alpha-1}e^{-x}/\Gamma(\alpha) \qquad (x > 0).$$

(i) Show that

$$f_\alpha * f_\beta = f_{\alpha+\beta}.$$

(ii) Deduce Euler's integral for the Beta function:

$$B(\alpha, \beta) := \int_0^1 x^{\alpha-1}(1-x)^{\beta-1}dx = \frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}.$$

Q6. For the shifted exponential distribution with parameter $\theta > 0$ (density $e^{-(x-\theta)}$ for $x > \theta$),
(i) find the MLE;
(ii) find for each n a sufficient statistic;
(iii) if θ has prior density $\lambda e^{-\lambda\theta}$ ($\theta \sim E(\lambda)$), find the posterior density to within a multiplicative constant.

NHB