smfprob5(14).tex

**SMF PROBLEMS 5. 14.11.2014**

Q1. In the regression model

$$y = A\beta + \epsilon$$

(data $y$ an $n$-vector, the design matrix $A$ an $n \times p$ matrix of constants, $\beta$ a $p$-vector of parameters, $\epsilon$ an $n$-vector of errors with independent $N(0, \sigma^2)$ components), show that the maximum-likelihood estimators, and also the least-squares estimators, are

$$\hat{\beta} = (A^T A)^{-1} A^T y.$$

Show also that (in the notation of lectures)

$$Py = A\hat{\beta}.$$

Q2. The $AR(p)$ process $(X_t)$ is given by

$$X_t = \phi_1 X_{t-1} + \cdots + \phi_p X_{t-p}, \qquad (\epsilon_t) \quad WN(\sigma^2).$$

(i) State without proof the condition for stationarity.
(ii) Derive the Yule-Walker equations for the autocorrelation $(\rho_k)$.
(iii) State the general solution of the Yule-Walker equations.

Q3. The $MA(1)$ process $(X_t)$ is given by

$$X_t = \epsilon_t + \theta\epsilon_{t-1}, \qquad |\theta| < 1, \qquad (\epsilon_t) \quad WN(\sigma^2).$$

Find
(i) the variance $\gamma_0 = var X_t$,
(ii) the autocovariance $\gamma_k = cov(X_t, X_{t+k})$,
(iii) the autocorrelation $\rho_k = corr(X_t, X_{t+k})$.

Q4. The time-series model is given by

$$X_t = X_{t-1} - \frac{1}{4} X_{t-2} + \epsilon_t + \frac{1}{2}\epsilon_{t-1}, \qquad (\epsilon_t) \quad WN(\sigma^2).$$

(i) Classify $(X_t)$ within the $ARIMA$ class.
(ii) Show that $(X_t)$ is stationary and invertible.

NHB