Practical Session 2

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Question 1: Preliminary analysis

Choose a stock of your choice and download its quotes from either Yahoo Finance or Google Finance. You will then need to transform stock prices into returns, for this we will use log-returns, i.e.:

$$\mu_{t_i} = \log\left(\frac{S_{t_{i+1}}}{S_{t_i}}\right), \quad i = 1, ..., N$$

where N is the number of data samples we have.

Perform, a preliminary statistical analysis on the data sample (for this you will find useful the commands *describe* in Python or *summary* in R) and briefly comment on the results.

A good practice when dealing with data and aiming to perform statistical testing is to compare it with a Gaussian sample. With this is mind, compare the histogram of your standardised sample (subtract the sample mean and normalise by the sample standard deviation) with a standard Gaussian density. Also, perform a Quantile-Quantile (Q-Q) plot. Comment the main differences that you observe. Is it sensible to assume that the population is normal in this case?

Question 2 : Higher order statistics

Compute higher order statistics on the data set, i.e. skewness and kurtosis (you will find useful the library *scipy.stats* in Python and the library *moments* in R). Note in particular that, the skewness is defined by:

$$Skew[X] = E\left[\left(\frac{X-\mu}{\sigma}\right)^3\right].$$

The kurtosis is defined by

$$Kurt[X] = E\left[\left(\frac{X-\mu}{\sigma}\right)^4\right].$$

The corresponding sample statistic for a population $x_1, ..., x_n$ is usually[1] computed by

$$\widehat{Skew} = \frac{\frac{1}{n}\sum_{i=1}^n (x_i - \overline{x})^3}{\sqrt{\frac{1}{n-1}\sum_{i=1}^n (x_i - \overline{x})^2}^3},$$

$$\widehat{Kurt} = \frac{\frac{1}{n}\sum_{i=1}^n (x_i - \overline{x})^4}{\left(\frac{1}{n-1}\sum_{i=1}^n (x_i - \overline{x})^2\right)^2}.$$

Compare the higher order statistics obtained in your sample with a Gaussian population and comment briefly your results.

Question 3: Hypothesis testing with trading strategies

Create a trading strategy that follows a simple rule (e.g. based on the average past returns buy or sell a stock) without any budget restrictions. Imagine that you have N agents and each of them will follow this rule on a different day, then you should get N samples of this strategy applied on N different days.

[1] You will find other sample statistics for the skewness and kurtosis in the literature, however for large samples all of them converge nicely

(a) Repeat again Question 1 with this new "trading" sample, and determine whether it is Gaussian.

(b) Next, you will need to test if the mean of the sample is equal to 0. For this, find a suitable statistic based on your results in part (a) (formally, you may need to make some further assumptions on the distribution of the sample in order to be able to find a statistic). Notice that the test we are interested in is given by:

$$H_0 : \mu = 0$$
$$H_1 : \mu \neq 0$$

Hint: Think about the distribution of $\bar{X}$ and your assumption on the distribution of your sample, as well as on σ^2.

(c) Imagine that you are an asset manager and your client is worried that this trading strategy loses money on average. Perform the following one-sided test:

$$H_0 : \mu \leq 0$$
$$H_1 : \mu > 0$$

and comment your findings.

(d) Also perform the following one-sided test:

$$H_0 : \mu \geq 0.$$
$$H_1 : \mu < 0$$

Comment your results. Does this test support your conclusion in (c) or is it inconsistent with part (c)?

Question 4 : Comparing trading strategies

Create another trading strategy with a different stock and test whether the mean of both strategies is equal:

$$H_0 : \mu_1 = \mu_2$$
$$H_1 : \mu_1 \neq \mu_2.$$

where μ_i represents the mean of the trading strategy i. For this, you will need to find a suitable statistic (formally, you may need to make some assumptions here as in Question 3.b).

Hint: Try to find the distribution of the difference of the means $\bar{X}_1 - \bar{X}_2$.