SMF SOLUTIONS 6. 2.3.2017

Q1 (*Brownian bridge*).

With the Brownian bridge defined as

$$B_0(t) : -B(t) - tB(1),$$

the mean is $E[B_0(t)] = E[B(t)] - tE[B(1)] = 0$. So the covariance is, for $s, t \in [0, 1]$ (as Brownian motion B has covariance $cov(B(s), B(t)) = E[B(s)B(t)] = \min(s, t)$)

$$\begin{aligned} cov(B_0(s), B_0(t)) &= E[B_0(s).B_0(t)] \\ &= E[(B(s) - sB(1))(B(t) - tB(1)] \\ &= E[B(s)B(t)] - tE[B(s)B(1)] - sE[B(t)B(1)] + stE[B(1)^2] \\ &= \min(s, t) - st - st + st \\ &= \min(s, t) - st. \end{aligned}$$

Q2 (*Median; breakdown point*).

For simplicity, take the sample size odd. The median is the point with half the data points below it. These can go off to $-\infty$ (and/or the points above can go off to $+\infty$) without dragging the median with them; but if more than half the points do this, they will drag the median with them. So, the median has breakdown point 1/2, as stated.

Q3 (*Quartiles; semi-inter-quartile range, SIQ*).

The lower quartile has a quarter of the data points beneath it. These can go off to $-\infty$ without dragging the lower quartile with them; but if more than a quarter do this, they will drag the lower quartile with them. So, the lower quartile has breakdown point 1/4. Similarly, so does the upper quartile. So the semi-interquartile range SIQ (half their difference) also has breakdown point 1/4.

NHB