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STATISTICAL METHODS FOR FINANCE: EXAMINATION 2013

Three hours; six questions, do five.

Q1. Define the (Fisher) *score function* $s(\theta)$ and the (Fisher) *information* $I(\theta)$.

Show that $s(\theta)$ has mean 0 and variance $I(\theta)$.

Describe briefly the use of $I(\theta)$ in estimation of parameters.

Q2. Define the *log-normal distribution* $LN(\mu, \sigma)$ with parameters μ and σ. Show that it has mean $\exp\{\mu + \frac{1}{2}\sigma^2\}$.

Describe briefly how the log-normal distribution occurs in mathematical finance.

For a normal distribution $N(\mu, \sigma^2)$ with σ known, obtain a uniformly most powerful test for the simple null hypothesis H_0: $\mu = \mu_0$ against the composite alternative hypothesis H_1: $\mu < \mu_0$.

Q3. Using either a classical or a Bayesian approach, as you may choose,
(i) define a *sufficient statistic* for a parameter θ;
(ii) state the (Fisher-Neyman) *factorisation criterion* for sufficiency;
(iii) prove their equivalence.

Q4. In a sample of size n, a response variable y has two linear predictor variables u and v. Construct the regression plane of y on u, v.

When is this plane unique?

Describe briefly the financial applications of this.

Q5. Define an *autoregressive* time-series model of *order* p, $AR(p)$.

Derive the *Yule-Walker equations*, and show how to solve them.

In the model

$$X_t = \frac{1}{3}X_{t-1} + \frac{2}{9}X_{t-2} + \epsilon_t, \qquad (\epsilon_t) \quad \text{white noise}, \tag{1}$$

find the autocorrelation function $\rho(k)$.

Q6. Define the *Bayes linear estimator* $a + b^T z$ for a parameter θ, and show that it is given by

$$d(z) = E\theta + cV^{-1}(z - Ez), \qquad c := cov(z, \theta), \quad V := var(z).$$

Describe briefly the applications of this result.

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