STATISTICAL METHODS FOR FINANCE: EXAMINATION, 2014.

Three hours; six questions – answer five.

Q1. In a parametric model, define the *score function* and the *information per reading*.

In a normal model with mean μ known,
(i) find the information per reading on the variance $v := \sigma^2$, and hence the Cramér-Rao bound;
(ii) find an efficient estimator for σ^2.

Q2. (i) Describe briefly the main contributions of Markowitz's work to mathematical finance.
(ii) Describe briefly the elliptically contoured model, and specify its parametric part and its non-parametric part.
(iii) What are the principal deficiencies of normal (Gaussian) models in mathematical finance?
(iv) Describe briefly how asset return distributions depend on the return period.

Q3. For a multivariate normal distribution $N(\mu, \Sigma)$, show that the maximum-likelihood estimators for μ, Σ are the sample mean $\bar{x}$ and the sample covariance matrix S. You may quote that the likelihood has the form ($V := \Sigma^{-1}$)

$$L = const.|V|^{n/2} \exp\{-\frac{1}{2}n\ trace(VS) - \frac{1}{2}n(\bar{x} - \mu)^T V(\bar{x} - \mu)\}.$$

Q4. For the regression model

$$y = A\beta + \epsilon,$$

(i) obtain the normal equations;
(ii) solve the normal equations for the parameter estimate $\hat{\beta}$;
(iii) show that

$$Py = A\hat{\beta},$$

where P is the projection matrix, which you should define.

Q5. For the $ARMA(1,1)$ model

$$X_t = \phi X_{t-1} + \epsilon_t + \theta \epsilon_{t-1}, \qquad \epsilon_t \sim N(0, \sigma^2),$$

give the condition for (a) stationarity, (b) invertibility.

Obtain

(i) the variance;

(ii) the covariance;

(iii) the autocorrelation.

Q6. For the normal distribution $N_n(\mu, \Sigma)$ in n dimensions, state without proof the density $f(x)$.

In a mixed-effects regression model

$$y|u \sim N(X\beta + Zu, R), \qquad u \sim N(0, D),$$

show that $u|y$ is multivariate normal,

$$u|y \sim N(\nu, \Sigma).$$

(You may quote Edgeworth's theorem and Bayes' formula without proof.)

Find ν and Σ.

Describe the occurrence of mixed-effects regression in finance.

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