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SMF EXAMINATION. 2011

Q1. (i) If $X \sim N_n(\mu, \Sigma)$, A and B are matrices, show that the linear forms AX, BX are independent iff

$$A\Sigma B^T = 0. \qquad \textbf{[8]}$$

(ii) If P is a symmetric projection and $\Sigma = \sigma^2 I$, show that
(a) the linear forms PX and $(I - P)X$ are independent; **[3]**
(b) the quadratic forms $X^T PX$ and $X^T(I - P)X$ are independent. **[7]**
(iii) If here P has rank k, find the distributions of these quadratic forms. **[7]**

Q2. The heights of two towers A and B are measured, and so is the difference of their heights (B is the higher). The true values are α and β; the readings are y_A, y_B, y_{B-A}. Formulate a regression model. **[6]**

Find
(a) the design matrix A, **[3]**
(b) $C := A^T A$, $C^{-1}A^T$, **[1,1]**
(c) the projection matrix $P := AC^{-1}A^T$ and $I - P$, **[1,1]**
(d) the parameter estimates, **[1,1]**
(e) the fitted values, **[1,1,1]**
(f) the ranks of P and $I - P$. **[1,1]**
(g) Describe briefly how to find an unbiased estimator of the error variance σ^2. **[5]**

Q3. (i) Define the *autoregressive model* of *order* p, $AR(p)$. Explain briefly, without proof, how to find the moving-average representation. **[5]**
(ii) Obtain the *Yule-Walker equations*, and explain briefly, without proof, how to solve them. **[5]**
(iii) Find the moving-average representation for the model

$$X_t = X_{t-1} - \frac{1}{4}X_{t-2} + \epsilon_t. \qquad \textbf{[10]}$$

(iv) Give, without proof, the senses in which your answer converges. **[5]**

Q4. A 3-vector x has the multivariate normal distribution $N(\mu, \Sigma)$, where

$$\Sigma = \begin{pmatrix} 1 & \rho & \rho \\ \rho & 1 & \rho \\ \rho & \rho & 1 \end{pmatrix}.$$

A 2-vector y is defined by $y_1 := x_1 + x_2$, $y_2 := x_2 + x_3$.
(i) Find the distribution of y. **[10]**
(ii) Find the conditional distribution of y_1 given y_2. **[15]**
(You may quote without proof the formula
$x_2|x_1 \sim N(\mu_2 + \Sigma_{21}\Sigma_{11}^{-1}(x_1 - \mu_1), \Sigma_{22} - \Sigma_{21}\Sigma_{11}^{-1}\Sigma_{12})$.)

Q5. Describe briefly, without proofs, the method of principal components analysis. **[6]**

Discuss the advantages and disadvantages of working with covariances and with correlations. **[6]**

Give examples, in the financial area, where each might be appropriate. **[13]**

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