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STATISTICAL METHODS FOR FINANCE: EXAMINATION 2012

Six questions; answer 5; 20 marks per question

Q1.(i) State without proof the Cramér-Rao inequality, and define an *efficient* estimator.
(ii) Outline, without proof, Fisher's method of scoring for the iterative solution of the likelihood equation.
(iii) Show how to apply the method of scoring to the Cauchy location family.

Q2. (i) In a parametric model, we have an estimator $T = T_n$ for the parameter θ, and an asymptotic result of the form

$$\sqrt{n}(T_n - \theta) \to N(0, \sigma^2(\theta)) \qquad (n \to \infty).$$

It is decided to reparametrise to

$$\phi := g(\theta),$$

with g continuously differentiable and increasing. Show how to transfer the result above for θ to the corresponding result for ϕ.
(ii) In a Bayesian setting, define the *Jeffreys prior*, and show that it is invariant under reparametrisation.
(iii) In a normal model $N(\mu, \sigma^2)$, compare the advantages and disadvantages of the variance σ^2 and the standard deviation σ as choice of second parameter.

Q3. In a multivariate setting, define the *sample mean vector* $\bar{x}$ and the *sample covariance matrix* $S = S_x$.

For the multivariate normal model $N(\mu, \Sigma)$,
(i) express the likelihood in terms of $\bar{x}$ and S;
(ii) show that $(\bar{x}, S)$ is sufficient for (μ, Σ).

Q4. In an $ARMA(1,1)$ model

$$X_t = \phi X_{t-1} + \epsilon_t + \theta \epsilon_{t-1},$$

(i) state (without proof) the conditions for stationarity and invertibility.
(ii) Assuming these, find
(a) the variance $\gamma_0 = var\ X_0$;
(b) the covariance $\gamma_k = cov(X_t, X_{t-k})$.

Q5. (i) For $x \sim N(\mu, \Sigma)$ multivariate normal, show that linear forms $u := Ax$, $v := Bx$ are independent if and only if $A\Sigma B^T = 0$.

In a regression model with $n \times p$ design matrix A, the *projection matrix* P is $P := A(A^T A)^{-1} A^T$.
(ii) Show that P, $I - P$ are projections.
(iii) Show that they have traces $tr(P) = p$, $tr(I - P) = n - p$.
(iv) Show that for an idempotent matrix, the eigenvalues are 0 or 1, and its trace is its rank.
(v) If P is a projection of rank r and x_i are independent $N(0, \sigma^2)$, show that the quadratic form $x^T P x$ is σ^2 times a $\chi^2(r)$-distributed random variable.

Q6. Describe briefly, without proofs, the method of principal components analysis.

Discuss the advantages and disadvantages of working with covariances and with correlations.

Give examples, in the financial area, where each might be appropriate.

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